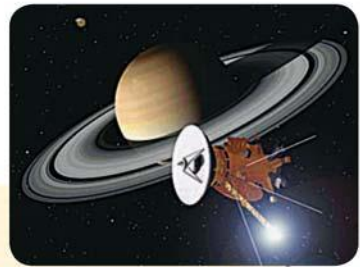
10.6 Find Segment Lengths in Circles



Before You found angle and arc measures in circles.

Now You will find segment lengths in circles.

Why? So you can find distances in astronomy, as in Example 4.

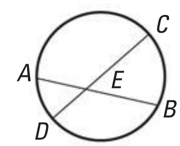
Key Vocabulary
- **segments of a chord**
- **secant segment**
- **external segment**

When two chords intersect in the interior of a circle, each chord is divided into two segments that are called **segments of the chord**.

THEOREM — *For Your Notebook*

THEOREM 10.14 Segments of Chords Theorem

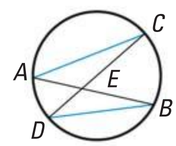
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$EA \cdot EB = EC \cdot ED$

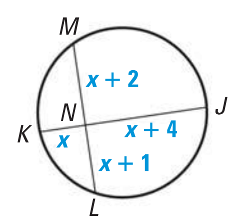
Proof: Ex. 21, p. 694

Plan for Proof To prove Theorem 10.14, construct two similar triangles. The lengths of the corresponding sides are proportional, so $\frac{EA}{ED} = \frac{EC}{EB}$. By the Cross Products Property, $EA \cdot EB = EC \cdot ED$.



EXAMPLE 1 Find lengths using Theorem 10.14

ALGEBRA **Find *ML* and *JK*.**



Solution

$NK \cdot NJ = NL \cdot NM$	**Use Theorem 10.14.**
$x \cdot (x + 4) = (x + 1) \cdot (x + 2)$	**Substitute.**
$x^2 + 4x = x^2 + 3x + 2$	**Simplify.**
$4x = 3x + 2$	**Subtract x^2 from each side.**
$x = 2$	**Solve for *x*.**

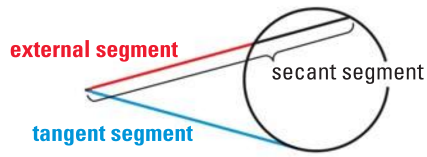
Find *ML* and *JK* by substitution.

$ML = (x + 2) + (x + 1)$ $\quad$ $JK = x + (x + 4)$

$= 2 + 2 + 2 + 1$ $\quad$ $= 2 + 2 + 4$

$= 7$ $\quad$ $= 8$

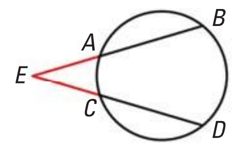
TANGENTS AND SECANTS A **secant segment** is a segment that contains a chord of a circle, and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.



THEOREM — *For Your Notebook*

THEOREM 10.15 Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



$EA \cdot EB = EC \cdot ED$

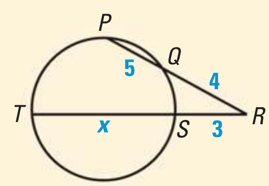
Proof: Ex. 25, p. 694

EXAMPLE 2 Standardized Test Practice

What is the value of x?

Ⓐ 6 Ⓑ $6\frac{2}{3}$

Ⓒ 8 Ⓓ 9



Solution

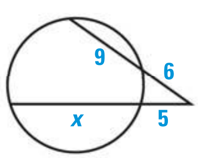
$RQ \cdot RP = RS \cdot RT$	Use Theorem 10.15.
$4 \cdot (5 + 4) = 3 \cdot (x + 3)$	Substitute.
$36 = 3x + 9$	Simplify.
$9 = x$	Solve for x.

▸ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

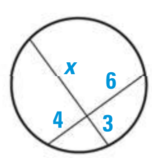
✓ GUIDED PRACTICE for Examples 1 and 2

Find the value(s) of x.

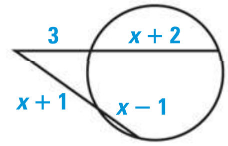
1.



2.



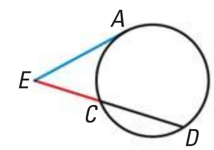
3.



THEOREM — *For Your Notebook*

THEOREM 10.16 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

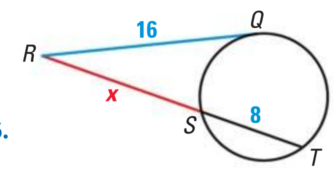


$EA^2 = EC \cdot ED$

Proof: Ex. 26, p. 694

EXAMPLE 3 Find lengths using Theorem 10.16

Use the figure at the right to find *RS*.



ANOTHER WAY

For an alternative method for solving the problem in Example 3, turn to page 696 for the **Problem Solving Workshop**.

Solution

$RQ^2 = RS \cdot RT$	**Use Theorem 10.16.**
$16^2 = x \cdot (x + 8)$	**Substitute.**
$256 = x^2 + 8x$	**Simplify.**
$0 = x^2 + 8x - 256$	**Write in standard form.**
$x = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(-256)}}{2(1)}$	**Use quadratic formula.**
$x = -4 \pm 4\sqrt{17}$	**Simplify.**

Use the positive solution, because lengths cannot be negative.

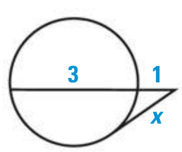
▶ So, $x = -4 + 4\sqrt{17} \approx 12.49$, and $RS \approx 12.49$.

Animated Geometry at classzone.com

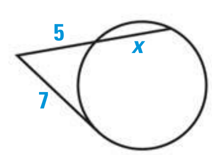
✓ GUIDED PRACTICE for Example 3

Find the value of *x*.

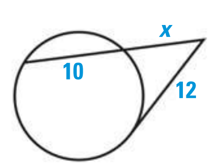
4.



5.

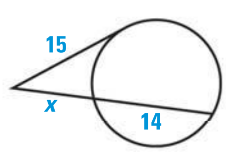


6.

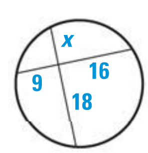


Determine which theorem you would use to find *x*. Then find the value of *x*.

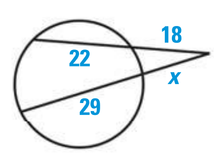
7.



8.



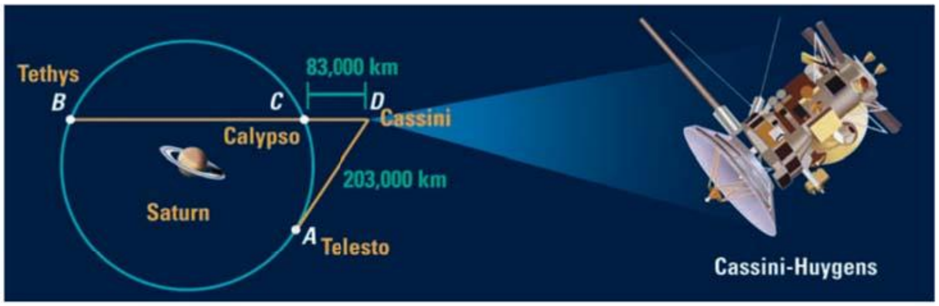
9.



10. In the diagram for Theorem 10.16, what must be true about *EC* compared to *EA*?

EXAMPLE 4 Solve a real-world problem

SCIENCE Tethys, Calypso, and Telesto are three of Saturn's moons. Each has a nearly circular orbit 295,000 kilometers in radius. The Cassini-Huygens spacecraft entered Saturn's orbit in July 2004. Telesto is on a point of tangency. Find the distance *DB* from Cassini to Tethys.



Solution

$DC \cdot DB = AD^2$	Use Theorem 10.16.
$83{,}000 \cdot DB \approx 203{,}000^2$	Substitute.
$DB \approx 496{,}494$	Solve for *DB*.

▶ Cassini is about 496,494 kilometers from Tethys.

✓ GUIDED PRACTICE for Example 4

11. Why is it appropriate to use the approximation symbol ≈ in the last two steps of the solution to Example 4?

10.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 9, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 24, and 27

SKILL PRACTICE

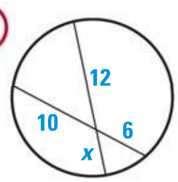
1. VOCABULARY Copy and complete: The part of the secant segment that is outside the circle is called a(n) _?_.

2. ★ WRITING *Explain* the difference between a tangent segment and a secant segment.

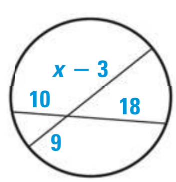
EXAMPLE 1 on p. 689 for Exs. 3–5

FINDING SEGMENT LENGTHS Find the value of *x*.

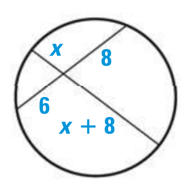
3.



4.



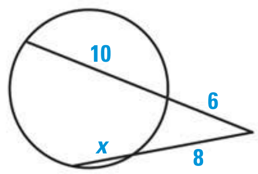
5.



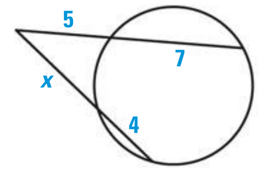
FINDING SEGMENT LENGTHS Find the value of x.

EXAMPLE 2
on p. 690
for Exs. 6–8

6.



7.



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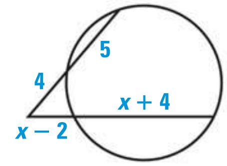
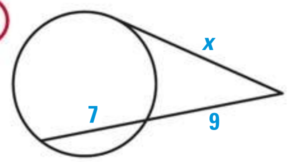
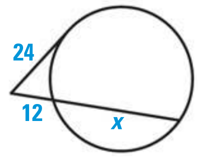


EXAMPLE 3
on p. 691
for Exs. 9–11

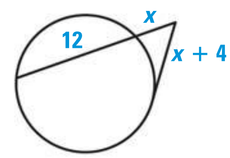
9.



10.

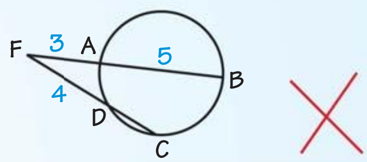


11.



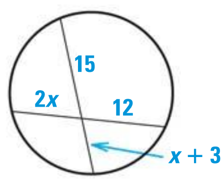
12. ERROR ANALYSIS *Describe* and correct the error in finding CD.

CD • DF = AB • AF
CD • 4 = 5 • 3
CD • 4 = 15
CD = 3.75

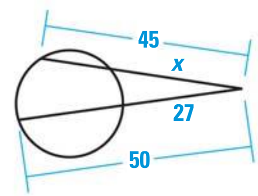


FINDING SEGMENT LENGTHS Find the value of x. Round to the nearest tenth.

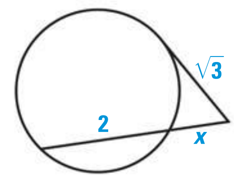
13.



14.

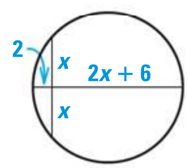


15.



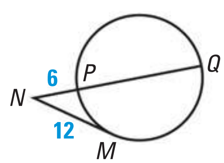
16. ★ MULTIPLE CHOICE Which of the following is a possible value of x?

Ⓐ −2 Ⓑ 4
Ⓒ 5 Ⓓ 6

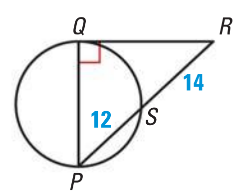


FINDING LENGTHS Find PQ. Round your answers to the nearest tenth.

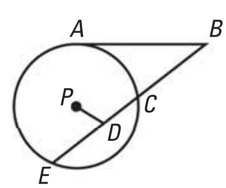
17.



18.



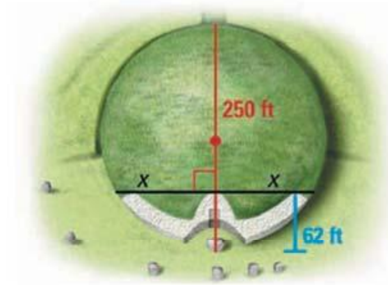
19. CHALLENGE In the figure, $AB = 12$, $BC = 8$, $DE = 6$, $PD = 4$, and A is a point of tangency. Find the radius of $\odot P$.



PROBLEM SOLVING

EXAMPLE 4
on p. 692
for Ex. 20

20. **ARCHAEOLOGY** The circular stone mound in Ireland called Newgrange has a diameter of 250 feet. A passage 62 feet long leads toward the center of the mound. Find the perpendicular distance x from the end of the passage to either side of the mound.



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21. **PROVING THEOREM 10.14** Write a two-column proof of Theorem 10.14. Use similar triangles as outlined in the Plan for Proof on page 689.

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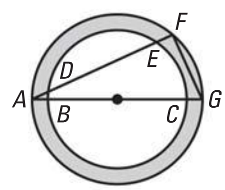
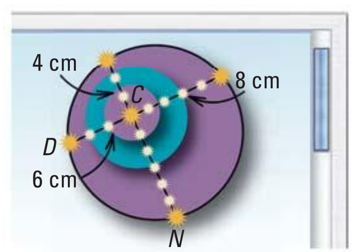
22. **WELLS** In the diagram of the water well, AB, AD, and DE are known. Write an equation for BC using these three measurements.



23. **PROOF** Use Theorem 10.1 to prove Theorem 10.16 for the special case when the secant segment contains the center of the circle.

24. ★ **SHORT RESPONSE** You are designing an animated logo for your website. Sparkles leave point C and move to the circle along the segments shown so that all of the sparkles reach the circle at the same time. Sparkles travel from point C to point D at 2 centimeters per second. How fast should sparkles move from point C to point N? *Explain.*

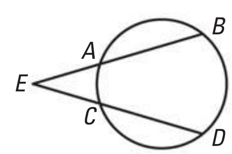


25. **PROVING THEOREM 10.15** Use the plan to prove Theorem 10.15.

GIVEN ▶ $\overline{EB}$ and $\overline{ED}$ are secant segments.

PROVE ▶ $EA \cdot EB = EC \cdot ED$

Plan for Proof Draw $\overline{AD}$ and $\overline{BC}$. Show that $\triangle BCE$ and $\triangle DAE$ are similar. Use the fact that corresponding side lengths in similar triangles are proportional.

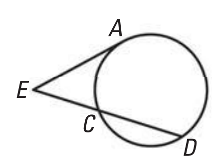


26. **PROVING THEOREM 10.16** Use the plan to prove Theorem 10.16.

GIVEN ▶ $\overline{EA}$ is a tangent segment.
$\overline{ED}$ is a secant segment.

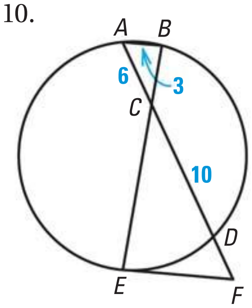
PROVE ▶ $EA^2 = EC \cdot ED$

Plan for Proof Draw $\overline{AD}$ and $\overline{AC}$. Use the fact that corresponding side lengths in similar triangles are proportional.



○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

27. ★ **EXTENDED RESPONSE** In the diagram, $\overline{EF}$ is a tangent segment, $m\overset{\frown}{AD} = 140°$, $m\overset{\frown}{AB} = 20°$, $m\angle EFD = 60°$, $AC = 6$, $AB = 3$, and $DC = 10$.



a. Find $m\angle CAB$.

b. Show that $\triangle ABC \sim \triangle FEC$.

c. Let $EF = y$ and $DF = x$. Use the results of part (b) to write a proportion involving x and y. Solve for y.

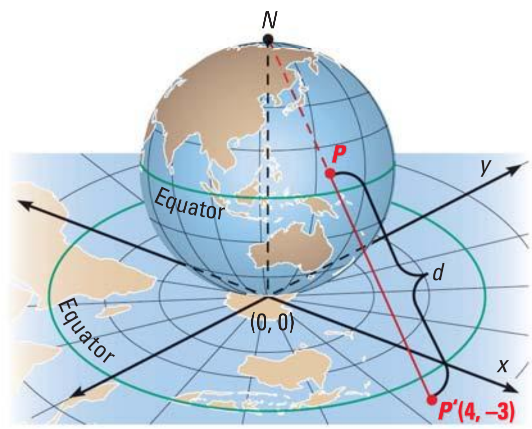
d. Use a theorem from this section to write another equation involving both x and y.

e. Use the results of parts (c) and (d) to solve for x and y.

f. *Explain* how to find CE.

28. **CHALLENGE** Stereographic projection is a map-making technique that takes points on a sphere with radius one unit (Earth) to points on a plane (the map). The plane is tangent to the sphere at the origin.

The map location for each point P on the sphere is found by extending the line that connects N and P. The point's projection is where the line intersects the plane. Find the distance d from the point P to its corresponding point $P'(4, -3)$ on the plane.



Not drawn to scale

MIXED REVIEW

PREVIEW
Prepare for Lesson 10.7 in Exs. 29–32.

Evaluate the expression. *(p. 874)*

29. $\sqrt{(-10)^2 - 8^2}$

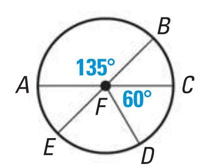
30. $\sqrt{-5 + (-4) + (6 - 1)^2}$

31. $\sqrt{[-2 - (-6)]^2 + (3 - 6)^2}$

32. In right $\triangle PQR$, $PQ = 8$, $m\angle Q = 40°$, and $m\angle R = 50°$. Find QR and PR to the nearest tenth. *(p. 473)*

33. $\overleftrightarrow{EF}$ is tangent to $\odot C$ at E. The radius of $\odot C$ is 5 and $EF = 8$. Find FC. *(p. 651)*

Find the indicated measure. $\overline{AC}$ and $\overline{BE}$ are diameters. *(p. 659)*



34. $m\overset{\frown}{AB}$

35. $m\overset{\frown}{CD}$

36. $m\overset{\frown}{BCA}$

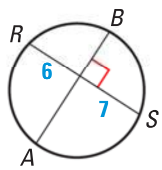
37. $m\overset{\frown}{CBD}$

38. $m\overset{\frown}{CDA}$

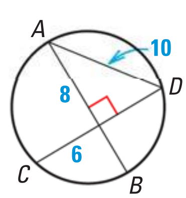
39. $m\overset{\frown}{BAE}$

Determine whether $\overline{AB}$ is a diameter of the circle. *Explain*. *(p. 664)*

40.



41.



42.